5 UPTHRUST IN FLUIDS, ARCHIMEDES' PRINCIPLE AND FLOATATION

Syllabus :

Buoyancy, Archimedes' principle, floatation, relationship with density; relative density, determination of relative density of a solid.

Scope : Buoyancy, upthrust (F_B); definition; different cases, F_B > = or < weight W of the body immersed; characteristic properties of upthrust; Archimedes' principle; explanation of cases where bodies with density ρ > = or < the density ρ' of the fluid in which it is immersed. R.D. and Archimedes' principle. Experimental determinations of R.D. of a solid and liquid denser than water. Floatation; principle of floatation; relation between the density of a floating body, density of the liquid in which it is floating and the fraction of volume of the body immersed; ($\rho_1/\rho_2 = V_2/V_1$); apparent weight of floating object; application to ship, submarine, iceberg, balloons, etc. Simple numerical problems involving Archimedes' principle and floatation.

(A) UPTHRUST AND ARCHIMEDES' PRINCIPLE

5.1 BUOYANCY AND UPTHRUST

When a body is partially or wholly immersed in a liquid, an upward force acts on it. This upward force is known as ***upthrust or buoyant force***. It is denoted by the symbol F_B. Thus

> The upward force exerted on a body by the fluid in which it is submerged, is called the upthrust or buoyant force.

The property of liquid to exert an upward force on a body immersed in it, is called ***buoyancy.*** This property can be demonstrated by the following experiments.

Exp. 1. Pushing an empty can into water : Take an empty can. Close its mouth with an airtight stopper. Put it in a tub filled with water. It floats with a large portion of it above the surface of water and only a small portion of it below the surface of water.

If we push the can into water, we feel an *upward force* which opposes the push and we find it difficult to push the can further into water. It is also noticed that as the can is pushed more and more into water, more and more force is needed to push the can further into water, *till it is completely immersed.* When the can is fully inside water, a constant force is still needed to keep it stationary in that position. Now if the can is released at this position, it is noticed that the can bounces back to the surface and starts floating again.

Exp. 2. Pushing a cork into water : If a piece of cork is placed on the surface of water in a tub, it floats with nearly $\frac{2}{5}$th of its volume inside water. If the cork is pushed into water and then released, it again comes to the surface of water and floats. If the cork is kept immersed, our fingers experience some upward force. The behaviour of cork is similar to that of the empty can.

Explanation : When the can or cork is put in the tub of water, *two* forces act on it : (i) its weight (*i.e.*, the force due to gravity) W which pulls it downwards, and (ii) the upthrust F_B due to water which pushes the can or cork upwards. It floats in the position when the two forces become equal in magnitude (*i.e.*, $W = F_B$). Now as the can or cork is pushed more and more inside water, the upthrust F_B exerted by water on it increases and becomes maximum ($= F'_B$) when it is completely immersed in water. So when it is released, the upthrust F'_B exerted by water on it being greater than its weight W (or force due to gravity), it rises up. To keep the can or cork immersed, an external downward force ($= F'_B - W$) is needed to balance the net upward force.

Note : *Like liquids, gases also have the property of buoyancy, i.e.,* a body immersed (or placed) in a gas also experiences an upthrust. All objects including ourselves, are also acted upon by a buoyant force due to air, but we do not feel it because it is negligibly small as compared to our own weight. On the other hand, a balloon filled with hydrogen (or any gas less denser than air) rises up because the upthrust (or buoyant force) on balloon due to the surrounding air is more than the weight of balloon filled with the gas.

Condition for a body to float or sink in a fluid : When a body is immersed in a fluid, *two* forces act on the body : (i) the weight W of

the body which acts vertically downwards and (ii) the upthrust F_B which acts vertically upwards. We have noticed that the upthrust depends on the submerged portion of the body. It increases as the submerged portion of body inside the fluid increases and becomes maximum $(= F'_B)$ when the body is completely immersed inside the fluid. Fig. 5.1 shows a body held completely immersed in a fluid with *two* forces W and F'_B acting on it.

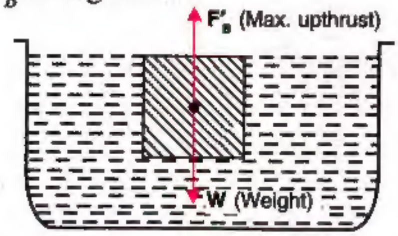

Fig. 5.1 Forces acting on a body held immersed inside a liquid

Depending upon the density of the fluid, the maximum buoyant force F'_B can be greater than, equal to or less than the weight W of the given body. Whether the body will float or sink in a fluid, depends on the relative magnitudes of forces W and F'_B (buoyant force when the body is fully immersed).

(i) If $F'_B > W$ or $F'_B = W$, the body will float (it will not sink). If $F'_B > W$, the body will float partly immersed with only that much part of it inside liquid, the upthrust F_B due to which becomes equal to the weight W of body (*i.e.*, $F_B = W$). But if $F'_B = W$, the body will float with whole of it immersed inside the liquid. Thus *for a floating body, net force acting downwards (i.e., apparent weight) is zero.*

(ii) If $F'_B < W$, the body will sink due to the net force $(W - F'_B)$ acting on the body downwards. If m is the mass of body, it will go down into the liquid with an acceleration a such that $ma = W - F'_B$ or $a = (W - F'_B)/m$. Here we have ignored the viscous force of the liquid.

Unit of upthrust : The upthrust, being a force, is measured in newton (N) or kgf.

5.2 CHARACTERISTIC PROPERTIES OF UPTHRUST

The upthrust has the following *three* characteristic properties :

(i) Larger the volume of body submerged in a fluid, greater is the upthrust.

(ii) For same volume inside the fluid more the density of fluid, greater is the upthrust.

(iii) The upthrust acts on the body in upward direction at the centre of buoyancy i.e., the centre of gravity of the displaced fluid.

(i) Larger the volume of body submerged in a fluid, greater is the upthrust

In the experiment of pushing an empty can or cork into water as described above, it is experienced that the upthrust on the body due to water increases as more and more volume of it is immersed into water, till it is completely immersed.

Similarly, when a bunch of feathers and a pebble of *same mass* are allowed to fall in air, the pebble falls faster than the bunch of feathers. The reason is that upthrust due to air on pebble is less than that on the bunch of feathers because the volume of pebble is less than that of the bunch of feathers of same mass. However in vacuum, both the bunch of feathers and pebble will fall together because there will be no upthrust.

(ii) For same volume inside the fluid more the density of fluid, greater is the upthrust

If we place a piece of cork A into water and another identical cork B into glycerine (or mercury), we notice that the volume of cork B immersed in glycerine (or mercury) is smaller as compared to the volume of cork A immersed in water. The reason is that the density of glycerine (or mercury) is more than that of water. Now if we want to immerse cork B in glycerine to the same extent as cork A in water, then an additional force is needed on cork B, to immerse it to the same level as cork A. This shows that for same volume of a body inside the liquid, a denser liquid exerts a greater upthrust.

(iii) The upthrust acts on the body in upward direction at the centre of buoyancy (i.e., the centre of gravity of the displaced liquid)

For a uniform body completely immersed inside a liquid, the centre of buoyancy coincides with the centre of gravity of the body (Fig. 5.1). But if a body floats in a liquid with its part submerged (Fig. 5.2), the centre of buoyancy B is at the centre of gravity of the displaced liquid (*i.e.*, at the centre of gravity of the immersed part

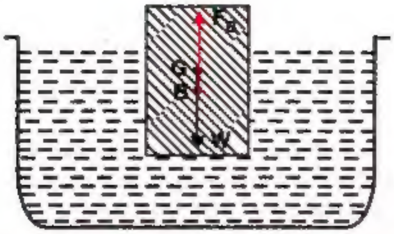

Fig. 5.2 A body floating with part of it submerged

of the body) which lies below the centre of gravity G of the entire body. The weight of the body W acts downwards at G, while upthrust F_B acts upwards at B such that $W = F_B$.

5.3 REASON FOR UPTHRUST

We have read that a liquid contained in a vessel exerts pressure at all points and in all directions. The pressure at a point in liquid is *same* in all directions (upwards, downwards and sideways). *It increases with depth inside the liquid.* When a body, say a block of area of cross section A, is immersed in a liquid (Fig. 5.3), the pressure P_2 exerted upwards on the lower face of block (which is at a greater depth) is *more* than the pressure P_1 exerted downwards on the upper face of block (which is at a lesser depth). Thus there is a difference in pressure $(= P_2 - P_1)$ between the lower and upper faces of block. Since force = pressure × area, the difference in pressures due to liquid on the two faces of block causes a net upward force (*i.e.*, upthrust) $= (P_2 - P_1)A$ on the body. However, the thrust on the side walls of body get neutralised as they are equal in magnitude and opposite in directions.

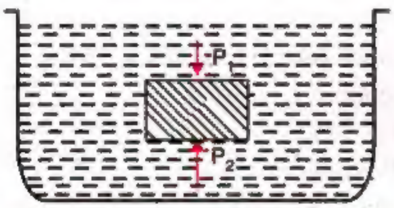

Fig. 5.3 A block immersed in a liquid

Note : If a lamina (thin sheet) is immersed in a liquid, the pressure on its both surfaces will be nearly same, so the liquid will exert negligible upthrust on it, causing it to sink into the liquid due to its own weight.

5.4 UPTHRUST IS EQUAL TO THE WEIGHT OF DISPLACED LIQUID (Mathematical proof)

When a body is immersed in a liquid, upthrust on it due to liquid is equal to the weight of the liquid displaced by the submerged part of the body.

Proof : Consider a cylindrical body $PQRS$ of cross-sectional area A immersed in a liquid of density ρ as shown in Fig. 5.4. Let the upper surface PQ of body be at a depth h_1 while its lower surface RS be at a depth h_2 below the free surface of liquid.

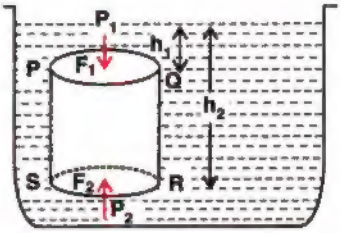

Fig. 5.4 Calculation for upthrust

At depth h_1, pressure on the upper surface PQ

$$P_1 = h_1 \rho g$$

∴ Downward thrust on the upper surface PQ

$$F_1 = \text{pressure} \times \text{area} = h_1 \rho g A \qquad ....(i)$$

At depth h_2, pressure on the lower surface RS

$$P_2 = h_2 \rho g$$

∴ Upward thrust on the lower surface RS

$$F_2 = h_2 \rho g A \qquad ....(ii)$$

The horizontal thrust at various points on the vertical sides of body get balanced because liquid pressure is same at all points at the same depth.

From above eqns. (i) and (ii), it is clear that $F_2 > F_1$ because $h_2 > h_1$ and therefore, the body will experience a net upward force.

Resultant upward thrust (or buoyant force) on the body

$$\begin{aligned} F_B &= F_2 - F_1 \\ &= h_2 \rho g A - h_1 \rho g A \\ &= A(h_2 - h_1)\,\rho g \end{aligned}$$

But $A(h_2 - h_1) = V$, the volume of the body submerged in liquid.

$$\therefore \qquad \boxed{\text{Upthrust } F_B = V \rho g} \qquad ....(5.1)$$

Since a solid when immersed in a liquid, displaces liquid equal to the volume of its submerged part, therefore

$V \rho g$ = Volume of solid immersed × density of liquid × acceleration due to gravity.

or $V\rho g$ = Volume of liquid displaced × density of liquid × acceleration due to gravity.

= mass of liquid displaced × acceleration due to gravity.

= Weight of the liquid displaced by the submerged part of the body.

Hence, **Upthrust = Weight of the liquid displaced by the submerged part of the body.** ...(5.2)

Note : **(1)** If the body is completely immersed in a liquid, the volume of liquid displaced will be equal to its own volume and upthrust then will be maximum ($= F'_B$).

(2) Although the above result is derived for a cylindrical body, but it is equally true for a body of any shape and size.

Factors affecting the upthrust

From the above discussion, it is clear that the magnitude of upthrust on a body due to a liquid (or fluid) depends on the following *two* factors :

(i) volume of the body submerged in liquid (or fluid), and

(ii) density of the liquid (or fluid) in which the body is submerged.

Effect of upthrust : *The effect of upthrust is that the weight of body immersed in a liquid appears to be less than its actual weight.* This can be demonstrated by the following experiment.

Experiment : Lifting of a bucket full of water from a well. Take an empty bucket and tie a long rope to it. If the bucket is immersed in water of a well keeping one end of rope in hand and the bucket is pulled when it is deep inside water, we notice that it is easy to pull the bucket as long as it is inside water, but as soon it starts coming out of the water surface, it appears to become heavy and now more force is needed to lift it.

This experiment shows that the bucket of water appears lighter when it is immersed in water than its actual weight (in air).

Similarly, when pulling a fish out of water, it appears lighter inside water as compared to when it is out of water.

Similarly, a body weighed by a sensitive spring balance, will weigh slightly less in air than in vacuum due to upthrust of air on the body.

5.5 ARCHIMEDES' PRINCIPLE

When a body is immersed in a liquid, it occupies the space, which was earlier occupied by the liquid *i.e.*, it displaces the liquid. *The volume of liquid displaced by the body is equal to the volume of the submerged part of the body so the body experiences an upthrust equal to the weight of the liquid displaced by it.*

It is the upthrust due to which a body immersed in a liquid appears to be of weight less than its real weight. *The apparent loss in weight is equal to the upthrust on the body.* This is called the Archimedes' principle. Thus

Archimedes' principle states that when a body is immersed partially or completely in a liquid, it experiences an upthrust, which is equal to the weight of the liquid displaced by it.

This principle applies not only to liquids, but it applies equally well to gases also.

5.6 EXPERIMENTAL VERIFICATION OF ARCHIMEDES' PRINCIPLE

Archimedes' principle can be verified by either of the following experiments.

Expt. (1) : Take two cylinders *A* and *B* of the same volume. The cylinder *A* is solid and the cylinder *B* is hollow. Suspend the two cylinders from the left arm of a physical balance keeping the solid cylinder *A* below the hollow cylinder *B*. Then balance the beam by keeping weights on right arm of the balance. In this situation, both cylinders *A* and *B* are in air.

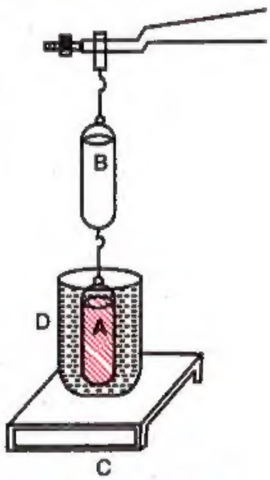

Fig. 5.5 Verification of Archimedes' principle

The solid cylinder *A* is now completely immersed into water contained in a beaker *D* placed on a bench *C* as shown in Fig. 5.5, taking care that the cylinder *A* does not touch the sides and bottom of the beaker. It is observed that the solid cylinder *A* loses weight *i.e.*, the left arm of the balance rises up. Obviously the loss in weight is due to upthrust (or buoyant force) of water on the cylinder *A*.

Now pour water gently in the hollow cylinder *B* till it is completely filled. It is observed that the beam balances again.

Thus, it is clear that *the buoyant force acting on solid cylinder A is equal to the weight of water*

filled in the hollow cylinder B. Since the cylinders A and B both have equal volume, so the weight of water in the hollow cylinder B is just equal to the weight of water displaced by the cylinder A. Hence the buoyant force acting on the cylinder A is equal to the weight of water displaced by it. Thus, it verifies the Archimedes' principle.

Expt. (2) : Take a solid (say, a metallic piece). Suspend it by a thin thread from the hook of a spring balance [Fig. 5.6(a)]. Note its weight.

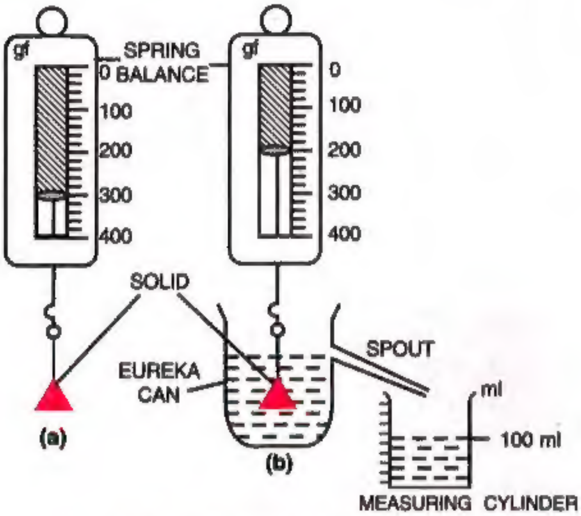

Fig. 5.6 Verification of Archimedes' principle

Now take a *eureka can* and fill it with water up to its spout. Arrange a measuring cylinder below the spout of the eureka can.

Now immerse the solid gently into water of the eureka can. The water displaced by it gets collected in the measuring cylinder [Fig. 5.6 (b)]. When water stops dripping through the spout, note the weight of the solid and the volume of water collected in the measuring cylinder.

In Fig. 5.6, the solid weighs 300 gf in air and 200 gf when it is completely immersed in water. The volume of water collected in the measuring cylinder is 100 ml *i.e.*, 100 cm^3.

$\therefore$ Loss in weight = 300 gf – 200 gf = 100 gf ....(i)

Volume of water displaced = Volume of solid = 100 cm^3

Since density of water = 1 g cm^{-3}

$\therefore$ Weight of water displaced = 100 gf ....(ii)

From eqns. (i) and (ii)

Weight of water displaced = Upthrust or loss in weight.

Thus the weight of water displaced by solid is equal to the loss in weight of the solid. This verifies Archimedes' principle.

5.7 SOLID BODIES WITH DENSITY (ρ) GREATER THAN DENSITY OF LIQUID (ρ_L) SINK WHILE WITH DENSITY (ρ) LESS THAN DENSITY OF LIQUID (ρ_L) FLOAT

Let a body of volume V and density ρ be immersed completely in a liquid of density ρ_L. The weight of the body acting downwards will be $W = V\rho g$ and the maximum upthrust on the body acting upwards will be $F'_B = V\rho_L g$. Following *three* cases may arise :

(i) If $W > F'_B$ or $V\rho g > V\rho_L g$ or $\rho > \rho_L$, the body will sink due to net force $(W - F'_B)$ acting downwards.

(ii) If $W = F'_B$ or $V\rho g = V\rho_L g$ or $\rho = \rho_L$, the body will float and the net force on the body is zero.

(iii) If $W < F'_B$ or $V\rho g < V\rho_L g$ or $\rho < \rho_L$, the body will float due to net force $(F'_B - W)$ acting upwards and only that much volume v of the body will submerg inside the liquid due to which upthrust F_B $(= v\rho_L g)$ balances the weight W. The net force on the body is zero in this situation also.

Thus a body of density ρ sinks in a liquid of density ρ_L if $\rho > \rho_L$, while it floats if $\rho = \rho_L$ or $\rho < \rho_L$. This can be demonstrated by the following experiments.

Expt. (1) : Take an iron nail and a piece of cork *both of same mass*. First place the iron nail on the surface of water contained in a cup. *The nail sinks*. It implies that the force of gravity (or weight) on iron nail pulling it downwards is greater than the upthrust of water on nail pushing it upwards. Now place the piece of cork on the surface of water. *The cork floats*. It means that upthrust on cork, when fully immersed is more than that on nail because the density of water is more than the density of cork, while the density of water is less than that of iron nail.

Expt. (2) : Take few solid bodies of different materials of known density and place them on the surface of water. It is observed that *if the*

density of the material of the body is equal to or *less than the density of water* (*i.e.*, $\rho = \rho_w$ or $\rho < \rho_w$), *it floats*, implying that the upthrust on body due to its submerged part is equal to its own weight (*i.e.*, $F_B = W$). Different bodies float on water with their different volumes inside water. If $\rho = \rho_w$, the body floats with whole of its volume inside water, while if $\rho < \rho_w$, the body floats with only that much volume inside water by which the upthrust F_B on body balances its weight W. On the other hand, *if the density of the material of body is more than the density of water* (*i.e.*, $\rho > \rho_w$), *the body sinks*, because the upthrust due to water on body is less than its weight (*i.e.*, $F_B < W$).

> Thus the *bodies of density greater than that of liquid, sink in it, while the bodies of average density equal to or smaller than that of liquid, float on it.*

An *empty* tin can (or iron ship) floats on water because its *average density** is less than the density of water.

* The average density of a hollow body is the ratio of mass of the body (= mass of material of body + mass of air enclosed) to its total volume.

EXAMPLES

1. A body weighs 200 gf in air and 190 gf when completely immersed in water. Calculate :

(i) the loss in weight of the body in water,

(ii) the upthrust on the body.

Given : Weight of the body in air = 200 gf

Weight of the body in water = 190 gf

(i) Loss in weight of the body = 200 gf − 190 gf = **10 gf**

(ii) Upthrust on the body = loss in weight = **10 gf.**

2. A small stone of mass m (= 200 g) is held under water in a tall jar and is allowed to fall as shown in Fig. 5.7. The forces acting on stone are also shown.

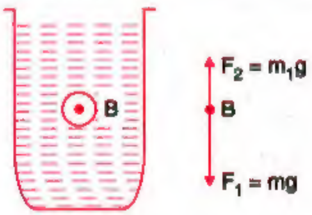

Fig. 5.7

(i) What does F_2 represent ?

(ii) What does m_1 represent ?

(iii) What is the net force acting on stone ?

(iv) What is the acceleration of stone as it falls through water ? Neglect the force due to viscosity. Assume that the volume of stone = 80 cm³, density of water = 1·0 g cm⁻³ and acceleration due to gravity g = 10 m s⁻².

(i) F_2 represents the **upthrust** on stone due to water.

(ii) m_1 represents the **mass of water displaced by stone.**

(iii) Net force acting on stone = $F_1 - F_2$ (downwards).

(iv) Given : $V = 80\ \text{cm}^3$, $\rho = 1\ \text{g cm}^{-3}$, $g = 10\ \text{m s}^{-2}$,

$$m = 200\ \text{g} = \frac{200}{1000}\ \text{kg} = 0{\cdot}2\ \text{kg}$$

∴ weight of stone $F_1 = mg = 0{\cdot}2\ \text{kg} \times 10\ \text{m s}^{-2} = 2\ \text{N}$

Mass of water displaced $m_1 = V\rho = 80 \times 1 = 80\ \text{g}$

$$= \frac{80}{1000}\ \text{kg} = 0{\cdot}08\ \text{kg}$$

upthrust $F_2 = m_1 g = 0{\cdot}08\ \text{kg} \times 10\ \text{m s}^{-2} = 0{\cdot}8\ \text{N}$

Hence net downward force on stone

$= F_1 - F_2$

$= 2 - 0{\cdot}8 = 1{\cdot}2\ \text{N}$

$$\therefore \text{Acceleration} = \frac{\text{Force}}{\text{Mass}} = \frac{1{\cdot}2\ \text{N}}{0{\cdot}2\ \text{kg}} = \mathbf{6\ m\ s^{-2}}.$$

3. A piece of iron of density $7{\cdot}8 \times 10^3$ kg m⁻³ and volume 100 cm³ is completely immersed in water (ρ = 1000 kg m⁻³). Calculate : (i) the weight of iron piece in air, (ii) the upthrust, and (iii) its apparent weight in water. (g = 10 m s⁻²)

Given : Volume of iron piece = 100 cm³

$= 100 \times 10^{-6}\ \text{m}^3 = 10^{-4}\ \text{m}^3$

(i) Weight of iron piece in air

= Volume × density of iron × g

$= 10^{-4} \times (7{\cdot}8 \times 10^3) \times 10$

= **7·8 N**

(ii) Upthrust = (Volume of water displaced) × density of water × g

But volume of water displaced = volume of iron piece when it is completely immersed = 10^{-4} m³

∴ Upthrust $= 10^{-4} \times 1000 \times 10 =$ **1 N**

(iii) Apparent weight = True weight – Upthrust

= 7·8 – 1 = **6·8 N.**

4. A metal cube of side 5 cm and density 7·9 g cm^{-3} is suspended by a thread and is immersed completely in a liquid of density 1·1 g cm^{-3}. Find : (a) the weight of cube, (b) the upthrust on cube and (c) the tension in thread.

(a) Given, side of cube = 5 cm

∴ Volume of the cube = 5 × 5 × 5 = 125 cm^3

Mass of the cube = Volume × density

= 125 cm^3 × 7·9 g cm^{-3}

= 987·5 g

∴ Weight of the cube = **987·5 gf (downwards)**

(b) Upthrust on cube= Weight of liquid displaced

= Volume of cube × density of liquid × g

= 125 × 1·1 × g = **137·5 gf (upwards)**

(c) Tension in thread = Net downward force

= Weight of cube – Upthrust on cube

= 987·5 – 137·5 = **850·0 gf.**

5. A solid of density ρ has weight *W*. Show that its apparent weight will be $W[1 - (\rho_L/\rho)]$ when it is completely immersed in a liquid of density ρ_L.

Given, weight of solid = W

∴ Mass of solid = W/g

$$\text{Volume of solid} = \frac{\text{Mass}}{\text{Density}} = \frac{W/g}{\rho}$$

Volume of liquid displaced = Volume of solid

$$= \frac{W/g}{\rho}$$

Upthrust on solid = Volume of liquid displaced × density of liquid × acceleration due to gravity

$$\text{or} \quad \text{upthrust} = \left(\frac{W/g}{\rho}\right) \times \rho_L \times g = \frac{W}{\rho} \times \rho_L$$

∴ Apparent weight = True weight – upthrust

$$= W - \left(\frac{W}{\rho} \times \rho_L\right) = W\left(1 - \frac{\rho_L}{\rho}\right)$$

Hence proved.

EXERCISE 5(A)

1. What do you understand by the term upthrust of a fluid ? Describe an experiment to show its existence.
2. In what direction and at what point does the buoyant force on a body due to a liquid, act ?
Ans. Upwards, at the centre of buoyancy.
3. What is meant by the term buoyancy ?
4. Define upthrust and state its S.I. unit.
5. What is the cause of upthrust ? At which point it can be considered to act ?
6. Why is a force needed to keep a block of wood inside water ?
Ans. Upthrust due to water on block when fully submerged is more than its weight.
7. A piece of wood if left under water, comes to the surface. Explain the reason.
8. Describe an experiment to show that a body immersed in a liquid appears lighter than it really is.
9. A metal solid cylinder tied to a thread is hanging from the hook of a spring balance. The cylinder is gradually immersed into water contained in a jar. What changes do you expect in the readings of spring balance ? Explain your answer.
10. Will a body weigh more in air or in vacuum when weighed with a spring balance ? Give a reason for your answer.
11. A body dipped into a liquid experiences an upthrust. State *two* factors on which upthrust on the body depends.
12. How is the upthrust related to the volume of the body submerged in a liquid ?
13. A bunch of feathers and a stone of the same mass are released simultaneously in air. Which will fall faster and why ? How will your observation be different if they are released simultaneously in vacuum ?
14. A body experiences an upthrust F_1 in river water and F_2 in sea water when dipped up to the same level. Which is more F_1 or F_2 ? Give reason.
Ans. $F_2 > F_1$. **Reason :** Sea water is denser than river water.
15. A small block of wood is held completely immersed in (i) water, (ii) glycerine and then released. In each case, what do you observe ? Explain the difference in your observation in the two cases.
16. A body of volume V and density ρ is kept completely immersed in a liquid of density ρ_L. If g is the acceleration due to gravity, write expressions for the following :
(i) the weight of the body,
(ii) the upthrust on the body,
(iii) the apparent weight of the body in liquid,
(iv) the loss in weight of the body.
Ans. (i) $V\rho g$ (ii) $V\rho_L g$ (iii) $V(\rho - \rho_L)g$ (iv) $V\rho_L g$
17. A body held completely immersed inside a liquid experiences two forces : (i) F_1, the force due to gravity and (ii) F_2, the buoyant force. Draw a diagram showing the direction of these forces

acting on the body and state condition when the body will float or sink.

18. Complete the following sentences :
    (a) Two balls, one of iron and the other of aluminium experience the same upthrust when dipped completely in water if ....
    (b) An empty tin container with its mouth closed has an average density equal to that of a liquid. The container is taken 2 m below the surface of that liquid and is left there. Then the container will ....
    (c) A piece of wood is held under water. The upthrust on it will be .... the weight of the wood piece.

Ans. (a) both have equal volume (b) will remains at the same position (c) more than.

19. Prove that the loss in weight of a body when immersed wholly or partially in a liquid is equal to the buoyant force (or upthrust) and this loss is because of the difference in pressure exerted by liquid on the upper and lower surfaces of the submerged part of body.

20. A sphere of iron and another of wood of the same radius are held under water. Compare the upthrust on the two spheres.
    [**Hint** : Both have equal volume inside water.]
    Ans. 1 : 1

21. A sphere of iron and another of wood, both of same radius are placed on the surface of water. State which of the two will sink ? Give reason to your answer. Ans. Sphere of iron will sink.
    Reason : $\rho_{iron} > \rho_{water}$, so weight of iron sphere will be more than upthrust due to water on it. But $\rho_{wood} < \rho_{water}$, so sphere of wood will float with its that much volume submerged inside water by which upthrust due to water on it balances its weight.

22. How does the density of material of a body determine whether it will float or sink in water ?

23. A body of density ρ is immersed in a liquid of density ρ_L. State condition when the body will (i) float, (ii) sink, in liquid.
    Ans. (i) $\rho <$ or $= \rho_L$ (ii) $\rho > \rho_L$

24. It is easier to lift a heavy stone under water than in air. Explain.

25. State Archimedes' principle.

26. Describe an experiment to verify the Archimedes' principle.

Multiple choice type :

1. A body will experience minimum upthrust when it is completely immersed in :
   (a) turpentine (b) water
   (c) glycerine (d) mercury.
   Ans. (a) turpentine

2. The S.I. unit of upthrust is :
   (a) Pa (b) N
   (c) kg (d) kg m^2
   Ans. (b) N

3. A body of density ρ sinks in a liquid of density ρ_L. The densities ρ and ρ_L are related as :
   (a) $\rho = \rho_L$ (b) $\rho < \rho_L$
   (c) $\rho > \rho_L$ (d) nothing can be said.
   Ans. (c) $\rho > \rho_L$

Numericals :

1. A body of volume 100 cm^3 weighs 5 kgf in air. It is completely immersed in a liquid of density $1{\cdot}8 \times 10^3$ kg m^{-3}. Find : (i) the upthrust due to liquid and (ii) the weight of the body in liquid.
   Ans. (i) 0·18 kgf (ii) 4·82 kgf

2. A body weighs 450 gf in air and 310 gf when completely immersed in water. Find :
   (i) the volume of the body,
   (ii) the loss in weight of the body, and
   (iii) the upthrust on the body.
   State the assumption made in part (i).
   Ans. (i) 140 cm^3 (ii) 140 gf (iii) 140 gf
   Assumption : density of water = 1·0 g cm^{-3}.

3. You are provided with a hollow iron ball *A* of volume 15 cm^3 and mass 12 g and a solid iron ball *B* of mass 12 g. Both are placed on the surface of water contained in a large tub. (a) Find upthrust on each ball. (b) Which ball will sink ? Give reason for your answer. (Density of iron = 8·0 g cm^{-3})
   Ans. (a) Upthrust on ball *A* = 12 gf and on ball *B* = 1·5 gf. (b) The ball *B* will sink.
   Reason : Volume of ball *B* = 12/8·0 = 1·5 cm^3. Upthrust on ball *B* is 1·5 gf which is less than its weight 12 gf, while upthrust on ball *A* will be 15 gf if it is fully submerged, which is greater than its weight 12 gf, so it will float with its that much part submerged for which upthrust becomes equal to its weight (=12 gf).

4. A solid of density 5000 kg m^{-3} weighs 0·5 kgf in air. It is completely immersed in water of density 1000 kg m^{-3}. Calculate the apparent weight of the solid in water. Ans. 0·4 kgf

5. Two spheres *A* and *B*, each of volume 100 cm^3 are placed on water (density = 1·0 g cm^{-3}). The sphere *A* is made of wood of density 0·3 g cm^{-3} and the sphere *B* is made of iron of density 8·9 g cm^{-3}.
   (a) Find : (i) the weight of each sphere, and (ii) the upthrust on each sphere.
   (b) Which sphere will float ? Give reason.
   Ans. (a) (i) ***A*** – 30 gf, ***B*** – 890 gf
   (ii) ***A*** – 30 gf, ***B*** – 100 gf
   (b) The sphere *A* will float
   Reason : The density of wood is less than the density of water.

6. The mass of a block made of a certain material is 13·5 kg and its volume is 15×10^{-3} m^3.
   (a) Calculate upthrust on the block if it is held fully immersed in water.
   (b) Will the block float or sink in water when released ? Give reason for your answer.
   (c) What will be the upthrust on block while floating? Take density of water = 1000 kg m^{-3}.

   Ans. (a) 15 kgf. (b) The block will float since upthrust on it when fully immersed in water, is more than its weight. (c) While floating, upthrust = 13·5 kgf.

7. A piece of brass weighs 175 gf in air and 150 gf when fully immersed in water. The density of water is 1·0 g cm^{-3}. (i) What is the volume of the brass piece ? (ii) Why does the brass piece weigh less in water ? Ans. (i) 25 cm^3 (ii) due to upthrust.

8. A metal cube of edge 5 cm and density 9·0 g cm^{-3} is suspended by a thread so as to be completely immersed in a liquid of density 1·2 g cm^{-3}. Find the tension in thread. (Take g = 10 m s^{-2})

   [**Hint:** Tension in thread = Apparent weight of the cube in liquid] Ans. 9·75 N

9. A block of wood is floating on water with its dimensions 50 cm × 50 cm × 50 cm inside water. Calculate the buoyant force acting on the block. Take g = 9·8 N kg^{-1}. Ans. 1225 N

10. A body of mass 3·5 kg displaces 1000 cm^3 of water when fully immersed inside it. Calculate : (i) the volume of body, (ii) the upthrust on body and (iii) the apparent weight of body in water.

    Ans. (i) 1000 cm^3 (ii) 1 kgf (iii) 2·5 kgf

(B) RELATIVE DENSITY AND ITS MEASUREMENT BY ARCHIMEDES' PRINCIPLE

5.8 DENSITY

If *equal volumes* of different substances such as wood, iron, zinc, water, glycerine, mercury etc., are weighed by a physical balance, we find that their *masses are different*. The mass of iron is much more than the mass of an equal volume of wood. This is because the particles of iron are heavier and more closely packed than those of wood. In other words, *iron is denser than wood.* In a similar manner, if we take *equal masses* of cotton and lead (say, one kg each), we notice that their *volumes are different*. The volume of cotton is much larger than the volume of an equal mass of lead. This is because the particles of lead are closely packed, while those of cotton are very loose. In other words, *lead is denser than cotton.* Thus to explain that equal volumes of different substances have different masses or equal masses of different substances have different volumes, we use a term called *density*. It is defined as follows :

The density of a substance is its mass per unit volume. i.e.,

Density of a substance

$$= \frac{\text{Mass of the substance}}{\text{Volume of the substance}} \quad \text{....(5.3)}$$

It is a **scalar** quantity and is represented by the letter ρ (rho) or *d*.

If mass of a substance is *M* and its volume is *V*, its density is

$$\boxed{\rho = \frac{M}{V}} \quad \text{...... (5.4)}$$

Unit of density

$$\text{Unit of density} = \frac{\text{Unit of mass}}{\text{Unit of volume}}$$

In S.I. system, unit of mass is kg and unit of volume is m^3, so S.I. unit of density is kg m^{-3}. In C.G.S. system, unit of mass is g and unit of volume is cm^3, so C.G.S. unit of density is g cm^{-3} (or gram per cubic centimetre).

Relationship between S.I. and C.G.S. units

$$1 \text{ kg m}^{-3} = \frac{1\text{kg}}{1\text{m}^3} = \frac{1000\text{ g}}{(100\text{ cm})^3}$$

$$= \frac{1}{1000} \text{ g cm}^{-3}$$

Thus

$$\boxed{\begin{array}{c} 1 \text{ kg m}^{-3} = 10^{-3} \text{ g cm}^{-3} \\ \text{or } \ 1 \text{ g cm}^{-3} = 1000 \text{ kg m}^{-3} \end{array}} \quad \text{...... (5.5)}$$

Example : The mass of 1 cm^3 of iron is 7·8 g, hence the density of iron is 7·8 g cm^{-3} or 7800 kg m^{-3}. Different substances have different densities.

Effect of temperature on density

Most of the substances expand on heating and contract on cooling, but their mass remains unchanged. Therefore, *density of most of the substances decreases with the increase in temperature and increases with the decrease in temperature.*

Exception : The behaviour of water is however very different due to its uneven expansion. Water when cooled from a high temperature, contracts up to 4°C thereafter it expands below 4°C up to 0°C. Thus the density of water gradually increases when it is cooled up to 4°C, and then starts decreasing when it is cooled further below 4°C up to 0°C. Thus,

> *The density of water is maximum at 4°C, equal to 1 g cm^{-3} or 1000 kg m^{-3}.*

5.9 RELATIVE DENSITY

[R.D. = $\frac{\rho_s}{\rho_w} = \frac{m_s}{m_w}$ FOR SAME VOLUME]

We have read that density of water at 4°C is 1 g cm^{-3} (or 1000 kg m^{-3}). Treating it as a standard, the density of a substance can be compared with the density of water at 4°C and the ratio so obtained is termed as the *relative density* of that substance. Thus,

> ***The relative density (R.D.) of a substance is the ratio of the density of that substance to the density of water at 4°C.***

i.e., $$\text{R.D.} = \frac{\text{Density of substance } (\rho_s)}{\text{Density of water at 4°C } (\rho_w)}$$

$$= \frac{\text{Mass of unit volume of substance}}{\text{Mass of unit volume of water at 4°C}}$$

$$= \frac{\text{Mass of substance } (m_s)}{\text{Mass of an equal volume of water at 4°C } (m_w)} \quad \text{... (5.6)}$$

Thus,

> ***Relative density of a substance is also defined as the ratio of the mass of substance to the mass of an equal volume of water at 4°C.***

Unit of relative density : Since relative density is a pure ratio, *it has no unit.* It is a **scalar quantity.**

Relationship between density and relative density : While calculating the relative density of a substance from its density (or density from its relative density), we note that

(i) In C.G.S. system, density of water at 4°C is 1 g cm^{-3}, so the relative density of a substance is equal to the numerical value of density of that substance. Thus

$$\text{R.D.} = \frac{\text{Density of substance in g cm}^{-3}}{1\cdot 0 \text{ g cm}^{-3}}$$

or $$\boxed{\text{Density in g cm}^{-3} = \text{R.D.}} \quad \text{....(5.7)}$$

(ii) In S.I. system, density of water at 4°C is 1000 kg m^{-3}, so its relative density is

$$\text{R.D.} = \frac{\text{Density of substance in kg m}^{-3}}{1000 \text{ kg m}^{-3}}$$

or $$\boxed{\text{Density in kg m}^{-3} = (\text{R.D.}) \times 1000} \quad \text{....(5.8)}$$

Examples :

(i) The density of copper is 8·9 g cm^{-3}, its R.D. is 8·9.

(ii) The density of mercury is 13·6 × 10^3 kg m^{-3}, its R.D. is 13·6.

(iii) The R.D. of silver is 10·8, its density in C.G.S. unit is 10·8 g cm^{-3} and in S.I. unit is 10·8 × 10^3 kg m^{-3}.

Difference between density and relative density

Density	*Relative density*
1. Density of a substance is the mass per unit volume of that substance.	1. Relative density of a substance is the ratio of density of that substance to the density of water at 4°C.
2. It is expressed in g cm^{-3} or kg m^{-3}.	2. It has no unit.

Density and R.D. of some common substances

Substance	*Density* kg m^{-3}	*Density* g cm^{-3}	*Relative density*
Cork	240	0·24	0·24
Wood (pine)	500	0·50	0·50
Petrol	800	0·80	0·80
Turpentine	870	0·87	0·87
Ice	920	0·92	0·92
Olive oil	920	0·92	0·92
Pure water (at 4°C)	1000	1·00	1
Sea water	1025	1·02	1·02
Glycerine	1260	1·26	1·26
Glass	2500	2·5	2·5
Aluminium	2700	2·70	2·70

Iron	7860	7·86	7·86
Copper	8920	8·92	8·92
Silver	10500	10·5	10·5
Mercury	13600	13·6	13·6
Gold	19300	19·3	19·3
Platinum	21500	21·5	21·5

5.10 DETERMINATION OF RELATIVE DENSITY OF A SOLID SUBSTANCE BY ARCHIMEDES' PRINCIPLE

We know that

$$\text{R.D.} = \frac{\text{Mass of the body}}{\text{Mass of water (at 4°C) of volume equal to that of the body}}$$

Using Archimedes' principle, the mass of water of volume equal to that of the body is obtained by finding the mass of water displaced by that body when it is completely immersed in water since a body when immersed in water, displaces water equal to its own volume. Therefore,

$$\text{R.D.} = \frac{\text{Mass of body}}{\text{Mass of water displaced by the body}}$$

$$= \frac{\text{Weight of body}}{\text{Weight of water displaced by the body}}$$

$$= \frac{\text{Weight of body}}{\text{Loss in weight of the body in water (or upthrust)}}$$

or $$\text{R.D.} = \frac{\text{Weight of body in air}}{\text{Weight of body in air} - \text{Weight of body in water}} \quad ...(5.9)$$

Thus, to find relative density of a solid body using Archimedes' principle, we have to weigh the body first in air and then in water. If the weight of body in air is W_1 and in water is W_2, then

$$\text{R.D.} = \frac{W_1}{W_1 - W_2} \quad ...(5.10)$$

Note : Weight and mass are related as weight = mass × acceleration due to gravity (*i.e.*, $W = Mg$). On weighing a body with a physical balance, its mass is expressed in kg or g, while its weight is expressed in kgf or gf.

Now we shall describe the procedure to determine the relative density of a solid in *two* cases : (i) when the solid is *denser* than water and *insoluble in it* and (ii) when the solid is *denser* than water and *soluble in it.*

(i) R.D. of a solid denser than water and insoluble in it

Procedure :

(i) Suspend a piece of the given solid with a thread from hook of the left pan of a physical balance and find its weight W_1.

(ii) Now place a wooden bridge over the left pan of balance and place a beaker nearly two-third filled with water on the bridge. Take care that the bridge and beaker do not touch the pan of balance.

(iii) Immerse the solid completely in water such that it does not touch the walls and bottom of beaker (Fig. 5.8) and find the weight W_2 of solid in water.

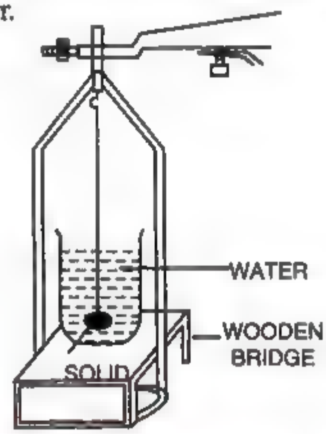

Fig. 5.8 R.D. of a solid denser than water

Observations :

Weight of solid in air $= W_1$ gf

Weight of solid in water $= W_2$ gf

Calculations :

Loss in weight of solid when immersed in water $= (W_1 - W_2)$ gf

$$\text{R.D.} = \frac{\text{Weight of solid in air}}{\text{Loss in weight of solid in water}}$$

or $$\text{R.D.} = \frac{W_1}{W_1 - W_2} \quad ....(5.11)$$

(ii) R.D. of a solid denser than water and soluble in it

Procedure : If solid is soluble in water, instead of water, we take a liquid of known relative density in which solid is insoluble and it sinks in that liquid. Then the process described above is repeated. Now

$$\text{R.D.} = \frac{\text{Weight of solid in air}}{\text{Loss in weight of solid in liquid}} \times \text{R.D. of liquid} \quad ....(5.12)$$

5.11 DETERMINATION OF RELATIVE DENSITY OF A LIQUID BY ARCHIMEDES' PRINCIPLE

By definition, relative density of a liquid is given as :

$$\text{R.D.} = \frac{\text{Weight of a given volume of the liquid}}{\text{Weight of the same volume of water}} \quad ...(5.13)$$

By Archimedes' principle if a solid is immersed in a liquid or water, it displaces the liquid or water equal to its own volume. Therefore the above eqn. (5.13) takes the form :

$$\text{R.D.} = \frac{\text{Weight of liquid displaced by a body}}{\text{Weight of water displaced by the same body}}$$

$$= \frac{\text{Weight of the body in air} - \text{weight of the body in liquid}}{\text{Weight of the body in air} - \text{weight of the body in water}} \quad ...(5.14)$$

Thus to find the relative density of a liquid using Archimedes' principle, we take a body which is heavier than both the given liquid and water and also insoluble in both. The body is first weighed in air, then in liquid and then after washing it with water and drying, it is weighed in water. If the weight of the body in air is W_1 gf, in liquid is W_2 gf and in water is W_3 gf, then from eqn. (5.14)

$$\boxed{\text{R.D. of liquid} = \frac{W_1 - W_2}{W_1 - W_3}} \quad ...(5.15)$$

EXAMPLES

1. Relative density of silver is 10·5. What is the density of silver in S.I. unit ? What assumption do you make in your calculations.

Given, R.D. of silver = 10·5

$$\text{R.D.} = \frac{\text{Density of silver}}{\text{Density of water}}$$

$\therefore$ Density of silver = R.D × density of water

$= \mathbf{10{\cdot}5 \times 10^3\ kg\ m^{-3}}$.

Assumption : Density of water = 10^3 kg m^{-3}.

2. A solid weighs 50 gf in air and 44 gf when completely immersed in water. Calculate :
(i) the upthrust,
(ii) the volume of the solid, and
(iii) the relative density of the solid.

Given, weight of solid in air W_1 = 50 gf and weight of solid in water W_2 = 44 gf.

(i) Upthrust = loss in weight when immersed in water = $W_1 - W_2$ = 50 – 44 = **6 gf**

(ii) Weight of water displaced = upthrust = 6 gf
Since density of water is 1 g cm^{-3}, therefore volume of water displaced = 6 cm^3
But a solid displaces water equal to its own volume, therefore volume of solid = **6 cm³**.

(iii) $$\text{R.D. of solid} = \frac{\text{Weight of solid in air}}{\text{Weight in air} - \text{Weight in water}}$$

$$= \frac{W_1}{W_1 - W_2} = \frac{50}{50-44} = \frac{50}{6} = \mathbf{8{\cdot}33}$$

3. A solid weighs 30 gf in air and 26 gf when completely immersed in a liquid of relative density 0·8. Find : (i) the volume of solid, and (ii) the relative density of solid.

Given, weight of solid in air W_1 = 30 gf and weight of solid in liquid W_2 = 26 gf., R.D. of liquid = 0·8

$\therefore$ Density of liquid = 0·8 g cm^{-3}

(i) Let V be the volume of solid.

Weight of liquid displaced = Volume of liquid displaced × density of liquid × g

$= V \times 0{\cdot}8 \times g$ dyne

$= V \times 0{\cdot}8$ gf ...(i)

Loss in weight of the solid when immersed in liquid

$= W_1 - W_2 = 30 - 26 = 4$ gf ...(ii)

But the weight of liquid displaced is equal to the loss in weight of solid when immersed in liquid.

$\therefore$ From eqns. (i) and (ii),

$\therefore$ $V \times 0{\cdot}8 = 4$

or $V = \dfrac{4}{0{\cdot}8} = \mathbf{5\ cm^3}$

(ii) Given, weight of solid = 30 gf

$\therefore$ Mass of solid = 30 g

$$\text{Density of solid} = \frac{\text{Mass}}{\text{Volume}} = \frac{30}{5} = 6\ \text{g cm}^{-3}.$$

Hence relative density of solid = **6**

Alternative method

(i) Volume of solid = Volume of liquid displaced
= mass of liquid displaced / density of liquid
= (30 – 26) / 0·8 = **5 cm³**.

(ii) R.D. of solid

$$= \frac{\text{Weight of solid in air}}{\text{Weight of solid in air} - \text{weight of solid in liquid}} \times \text{R.D. of liquid}$$

$$= \frac{30}{30-26} \times 0{\cdot}8 = \frac{30}{4} \times 0{\cdot}8 = \mathbf{6}$$

4. A solid body weighs 2·10 N in air. Its relative density is 8·4. How much will the body weigh if placed
(i) in water,
(ii) in a liquid of relative density 1·2 ?

(i) Given : Weight of the body in air W_1 = 2·10 N, R.D. of body = 8·4, weight of body in water W_2 = ?

$$\text{R.D.} = \frac{W_1}{W_1 - W_2} \quad \therefore \quad 8\cdot4 = \frac{2\cdot1}{2\cdot1 - W_2}$$

or $8\cdot4\,(2\cdot1 - W_2) = 2\cdot1$

or $$W_2 = \frac{2\cdot1 \times 7\cdot4}{8\cdot4} = \mathbf{1\cdot85\ N}$$

Thus weight of body in water = **1·85 N**

(ii) Upthrust due to water = $W_1 - W_2$ = 2·10 – 1·85 = 0·25 N

Upthrust due to liquid
= Upthrust due to water × R.D. of liquid
= 0·25 × 1·2 = 0·30 N

∴ Weight of body in liquid = Weight of body in air – Upthrust due to liquid
= 2·10 – 0·30 = **1·8 N.**

Alternative method :

Let weight of body in liquid be x N. Then R.D.

$$= \frac{\text{Weight of body in air}}{\text{Weight of body in air} - \text{Weight of body in liquid}} \times \text{R.D. of liquid}$$

or $$8\cdot4 = \frac{2\cdot1}{2\cdot1 - x} \times 1\cdot2$$

or $$4\,(2\cdot1 - x) = 1\cdot2 \quad \text{or} \quad x = \frac{7\cdot2}{4} = \mathbf{1\cdot8\ N}$$

5. A body weighs 82·1 gf in air, 75·5 gf in water and 73·8 gf in a liquid. (a) Find the relative density of the liquid. (b) How much will it weigh if immersed in a liquid of relative density 0·87 ?

(a) Given, weight of the body in air W_1 = 82·1 gf

Weight of the body in liquid = W_2 = 73·8 gf

Weight of the body in water W_3 = 75·5 gf

$$\text{R.D. of liquid} = \frac{W_1 - W_2}{W_1 - W_3}$$

$$= \frac{82\cdot1 - 73\cdot8}{82\cdot1 - 75\cdot5} = \frac{8\cdot3}{6\cdot6} = \mathbf{1\cdot26}$$

(b) Given, R.D. of liquid = 0·87, W_1 = 82·1 gf, W_2 = ?, W_3 = 75·5 gf

From relation $$\text{R.D.} = \frac{W_1 - W_2}{W_1 - W_3}$$

$$0\cdot87 = \frac{82\cdot1 - W_2}{82\cdot1 - 75\cdot5}$$

or $82\cdot1 - W_2 = 0\cdot87 \times 6\cdot6 = 5\cdot742$

∴ $W_2 = 82\cdot1 - 5\cdot742 = 76\cdot358$

$= \mathbf{76\cdot4\ gf}$

EXERCISE 5(B)

1. Define the term density.
2. What are the units of density in (i) C.G.S. and (ii) S.I. system. **Ans.** (i) g cm^{-3} (ii) kg m^{-3}
3. Express the relationship between the C.G.S. and S.I. units of density. **Ans.** 1 g cm^{-3} = 1000 kg m^{-3}
4. 'The density of iron is 7800 kg m^{-3}'. What do you understand by this statement ?
5. Write the density of water at 4°C in S.I. unit. **Ans.** 1000 kg m^{-3}
6. How are the (i) mass, (ii) volume, and (iii) density of a metallic piece affected, if at all, with increase in temperature ? **Ans.** (i) unchanged, (ii) increases, (iii) decreases.
7. Water is heated from 0°C to 10°C. How does the density of water change with temperature ? **Ans.** On heating from 0°C, the density of water increases up to 4°C and then decreases beyond 4°C.
8. Complete the following sentences :
   (i) Mass = …….. × density
   (ii) S.I. unit of density is …….
   (iii) Density of water is ... kg m^{-3}.
   (iv) Density in kg m^{-3} = … × density in g cm^{-3}
   Ans. (i) volume, (ii) kg m^{-3}, (iii) 1000, (iv) 1000
9. What do you understand by the term relative density of a substance ?
10. What is the unit of relative density ? **Ans.** No unit
11. Differentiate between density and relative density of a substance.
12. With the use of Archimedes' principle, state how you will find relative density of a solid denser than water and insoluble in it. How will you modify your experiment if the solid is soluble in water ?
13. A body weighs W gf in air and W_1 gf when it is completely immersed in water. Find : (i) volume

of the body, (ii) upthrust on the body, (iii) relative density of material of the body.

Ans. (i) $(W - W_1)$ cm^3 (ii) $(W - W_1)$ gf (iii) $\frac{W}{W - W_1}$

14. Describe an experiment, using Archimedes' principle, to find relative density of a liquid.

15. A body weighs W_1 gf in air and when immersed in a liquid it weighs W_2 gf, while it weighs W_3 gf on immersing it in water. Find : (i) volume of the body (ii) upthrust due to liquid (iii) relative density of the solid and (iv) relative density of the liquid.

Ans. (i) $(W_1 - W_3)$ cm^3, (ii) $(W_1 - W_2)$ gf, (iii) $\frac{W_1}{W_1 - W_3}$ (iv) $\frac{W_1 - W_2}{W_1 - W_3}$

Multiple choice type :

1. Relative density of a substance is expressed by comparing the density of that substance with the density of :
   (a) air (b) mercury
   (c) water (d) iron.
   Ans. (c) water

2. The unit of relative density is :
   (a) g cm^{-3} (b) kg m^{-3}
   (c) m^3 kg^{-1} (d) no unit.
   Ans. (d) no unit

3. The density of water is :
   (a) 1000 g cm^{-3} (b) 1 kg m^{-3}
   (c) 1 g cm^{-3} (d) none of these.
   Ans. (c) 1 g cm^{-3}

Numericals :

1. The density of copper is 8·83 g cm^{-3}. Express it in kg m^{-3}. **Ans.** 8830 kg m^{-3}

2. The relative density of mercury is 13·6. State its density in (i) C.G.S. unit, (ii) S.I. unit.
   Ans. (i) 13·6 g cm^{-3} (ii) 13·6 × 10^3 kg m^{-3}

3. The density of iron is 7·8 × 10^3 kg m^{-3}. What is its relative density ? **Ans.** 7·8

4. The relative density of silver is 10·8. Find its density. **Ans.** 10·8 × 10^3 kg m^{-3}

5. Calculate the mass of a body whose volume is 2 m^3 and relative density is 0·52. **Ans.** 1040 kg

6. Calculate the mass of air in a room of dimensions 4·5 m × 3·5 m × 2·5 m if the density of air at N.T.P. is 1·3 kg m^{-3}. **Ans.** 51·19 kg

7. A piece of stone of mass 113 g sinks to the bottom in water contained in a measuring cylinder and water level in cylinder rises from 30 ml to 40 ml. Calculate R.D. of stone. **Ans.** 11·3

8. A body of volume 100 cm^3 weighs 1 kgf in air. Find : (i) its weight in water and (ii) its relative density.
   Ans. (i) 900 gf, (ii) 10

9. A body of mass 70 kg, when completely immersed in water, displaces 20,000 cm^3 of water. Find : (i) the weight of body in water and (ii) the relative density of material of body.
   Ans. (i) 50 kgf, (ii) 3·5

10. A solid weighs 120 gf in air and and 105 gf when it is completely immersed in water. Calculate the relative density of solid.
    Ans. 8

11. A solid weighs 32 gf in air and 28·8 gf in water. Find : (i) the volume of solid, (ii) R.D. of solid, and (iii) the weight of solid in a liquid of density 0·9 g cm^{-3}.
    Ans. (i) 3·2 cm^3, (ii) 10, (iii) 29·12 gf

12. A body weighs 20 gf in air and 18·0 gf in water. Calculate relative density of the material of body.
    Ans. 10

13. A solid weighs 1·5 kgf in air and 0·9 kgf in a liquid of density 1·2 × 10^3 kg m^{-3}. Calculate R.D. of solid. **Ans.** 3·0

14. A jeweller claims that he makes ornament of pure gold of relative density 19·3. He sells a bangle weighing 25·25 gf to a person. The clever customer weighs the bangle when immersed in water and finds that it weighs 23·075 gf in water. With the help of suitable calculations find out whether the ornament is made of pure gold or not.

    [**Hint :** Calculate R.D. of material of bangle which comes out to be 11·6].

    Ans. Gold is not pure.

15. A piece of iron weighs 44·5 gf in air. If the density of iron is 8·9 × 10^3 kg m^{-3}, find the weight of iron piece when immersed in water.
    Ans. 39·5 gm

16. A piece of stone of mass 15·1 g is first immersed in a liquid and it weighs 10·9 gf. Then on immersing the piece of stone in water, it weighs 9·7 gf. Calculate :
    (a) the weight of the piece of stone in air,
    (b) the volume of the piece of stone,
    (c) the relative density of stone,
    (d) the relative density of the liquid.
    Ans. (a) 15·1 gf, (b) 5·4 cm^3, (c) 2·8, (d) 0·78

5.12 PRINCIPLE OF FLOATATION

We have read that when a body is immersed in a liquid, the following *two* forces act on it :

(i) The weight W of body acting vertically downwards, through the centre of gravity G of the body. This force has a tendency to sink the body.

(ii) The upthrust F_B of the liquid acting vertically upwards, *through the centre of buoyancy B i.e., the centre of gravity of the displaced liquid*. The upthrust (or buoyant force) is equal in magnitude to the weight of the liquid displaced. This force has a tendency to make the body float.

Fig. 5.9 shows the two forces W and F_B acting on a body floating on a liquid.

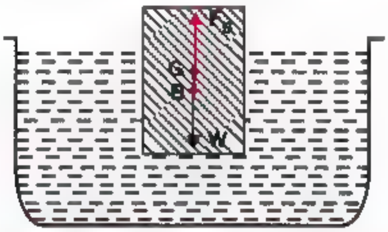

Fig. 5.9 Two forces acting on a floating body

In magnitude,

W = volume of body × density of body × g ..(5.16)

and F_B = volume of submerged part of body × density of liquid × g ...(5.17)

Obviously, the upthrust F_B is maximum ($=F'_B$) when the body is completely immersed inside the liquid.

Depending upon whether the maximum upthrust F'_B is less than, equal to or greater than the weight W, the body will either sink or float in liquid. So we consider the following *three* cases when (i) $W > F'_B$ (ii) $W = F'_B$ and (iii) $W < F'_B$.

Case (i) : *When $W > F'_B$ i.e., the weight of the body is greater than the weight of the displaced liquid.* In this case, the body will sink as shown in Fig. 5.10. The apparent weight of the body (*i.e.*, the weight of body inside liquid) as measured by a spring balance if it is attached with the body will be $(W - F'_B)$ acting vertically downwards. This is the case when the density ρ of solid is greater than the density ρ_L of liquid (*i.e.*, $\rho > \rho_L$).

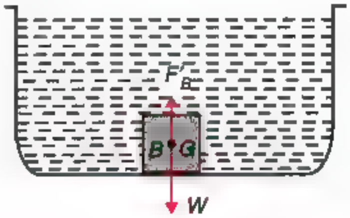

Fig. 5.10 The body sinks when $W > F'_B$

Case (ii) : *When $W = F'_B$ i.e., the weight of the body is equal to the weight of the displaced liquid.* In this case, the body will float just below the surface of liquid as shown in Fig. 5.11. The *apparent weight of body will be zero*. The density ρ of such a body is equal to the density ρ_L of liquid (*i.e.*, $\rho = \rho_L$).

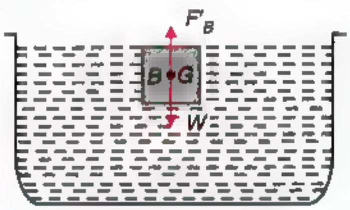

Fig. 5.11 The body floats when $W = F'_B$

Case (iii) : *When $W < F'_B$ i.e., the weight of the body is less than the weight of the liquid displaced by it when it is held completely immersed in the liquid.* In this case, the body floats partially above and partially below the surface of liquid as shown in Fig. 5.12. *Only that much portion of the body gets submerged by which the weight of displaced liquid becomes equal to the weight of the body*. In this situation,

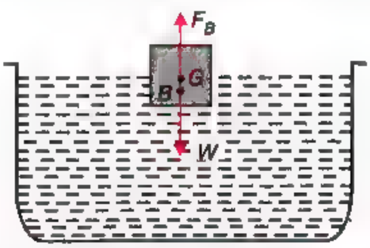

Fig. 5.12 The body floats partially inside the liquid when $W < F'_B$ on complete immersion

while floating, weight W acts at the centre of gravity G of the body, while upthrust F_B acts at the centre of buoyancy B which is vertically below G, and weight W is equal to upthrust F_B *only due to the submerged part of the body*. Thus, the apparent weight of the body will be zero in this case also. The density ρ of such a body is less than the density ρ_L of liquid (*i.e.*, $\rho < \rho_L$).

From the above discussion, we find that for a floating body,

> Weight of body
> = Weight of liquid displaced by the submerged part of body.
> or $W = F_B$ .....(5.18)
> $\therefore$ Apparent weight of a floating body = 0 (zero)

This is the *principle of floatation*. Thus,

> ***According to the principle of floatation, the weight of a floating body is equal to the weight of the liquid displaced by its submerged part.***

5.13 RELATION BETWEEN VOLUME OF SUBMERGED PART OF A FLOATING BODY, THE DENSITIES OF LIQUID AND THE BODY $\left(\frac{v}{V} = \frac{\rho_s}{\rho_L}\right)$

Let V be the volume of a body of density ρ_s. Let the body be floating with its volume v immersed inside a liquid of density ρ_L. Then

Weight of the body W = Volume of body × density of body × g

$= V\rho_s g$

Weight of liquid displaced by the body or upthrust

F_B = Volume of displaced liquid × density of liquid × $g = v\rho_L g$

For floatation, $W = F_B$

i.e., $V\rho_s g = v\rho_L g$

or $$\boxed{\frac{v}{V} = \frac{\rho_s}{\rho_L}} \quad .....(5.19)$$

Thus,

$$\frac{\text{Volume of immersed part of body}}{\text{Total volume of body}} = \frac{\text{Density of body}}{\text{Density of liquid}}$$

Examples : (1) A cork of density $\rho_s = \frac{3}{4}$ g cm^{-3} while floating in water will have its $\frac{3}{4}$th part immersed inside water (ρ_L = 1 g cm^{-3}) and $\frac{1}{4}$th part outside the surface of water.

(2) A cube of ice (density ρ_s = 0·9 g cm^{-3}) will have 90% of its volume immersed in water (density ρ_L = 1 g cm^{-3}) while floating and only 10% outside the surface of water.

5.14 APPLICATIONS OF THE PRINCIPLE OF FLOATATION

(i) *Floatation of iron ship*

An iron nail sinks in water while a ship floats : If we place an iron nail on the surface of water, it sinks. This is because the density of iron is greater than that of water, so the weight of nail is more than the upthrust of water on it.

On the other hand, ships are also made of iron, but they do not sink. This is because the ship is hollow and the empty space in it contains air which makes its volume large and *average density less than that of water*. Therefore, even with a small portion of ship submerged in water, the weight of water displaced by the submerged part of ship becomes equal to the total weight of ship and therefore it floats.

A loaded ship is submerged more while an unloaded ship is less submerged : When cargo is loaded on a sailing ship, its weight increases, so it sinks further to displace more water till the weight of water displaced by its submerged part becomes equal to the weight of loaded ship. If cargo is unloaded, the ship will rise in water till the weight of water displaced balances the weight of unloaded ship.

A ship begins to submerge more as it sails from sea water to river water : The water of river is of low density than that of a sea and the density of water of different sea is also different. Therefore, when a ship sails from a sea of water of higher density to a sea of water of lower density (or from sea water to river water), it sinks further. The reason is that according to the law of floatation, to balance the weight of ship,

a greater volume of water is required to be displaced in water of lower density in river (or sea).

Plimsoll line : Each ship has a white line painted on its side, known as the *Plimsoll line*. This line indicates the safe limit for loading the ship in water of density 10^3 kg m^{-3}. A ship is not allowed to be loaded further when its Plimsoll line starts touching the water level, so that when it sails in sea water of density more than 10^3 kg m^{-3}, only the part of it below the plimsoll line remains submerged in water.

An unloaded ship is filled with sand at its bottom : An unloaded ship floats with its very small volume inside water. As a result, its centre of gravity is higher and its equilibrium is *unstable*. There is a danger that it may get blown over on its side by strong winds. Therefore, an unloaded ship is filled with sand (or stones), called *ballast*, at its bottom. This lowers its centre of gravity to make its equilibrium stable.

(ii) *Floatation of human body*

The average density of human body depends on the proportion of its constituents like bone, blood, muscles and fat in him as each constituent has different density. Further, it also depends on the amount of air in his lungs at that time. The average density of body with empty lungs is 1·07 g cm^{-3}, while with lungs filled with air is 1·00 g cm^{-3}. A good swimmer can float on water, like a floating object, with his lungs filled with air and nose and mouth projecting just above the water surface. The weight of water displaced by him is then nearly equal to his own weight. Thus, he can swim with a very little effort.

It is easier for a man to swim in sea water than in fresh (or river) water : The reason is that due to presence of minerals (salt etc.), the density of sea water (= 1·026 g cm^{-3}) is more than the density of fresh (or river) water (= 1·0 g cm^{-3}). Therefore, with a smaller portion of the body submerged in sea water, the weight of water displaced becomes equal to the total weight of the body, while to displace the same weight of fresh (or river) water, a large portion of his body will have to be submerged in water. So it becomes difficult to swim in river water.

In the Dead Sea, the density of water is much more (= 1·16 g cm^{-3}), therefore, a man can easily swim in Dead Sea with a small portion submerged inside water so as to balance his weight.

(iii) *Floatation of submarines*

A submarine is a fish shaped water-tight boat provided with several ballast (or *floatation*) tanks in its front and rear parts. Fig. 5.13 shows the portion of a submarine to explain its floatation. It is provided with periscopes so that the diver could see above the water surface even when the submarine is well inside the water.

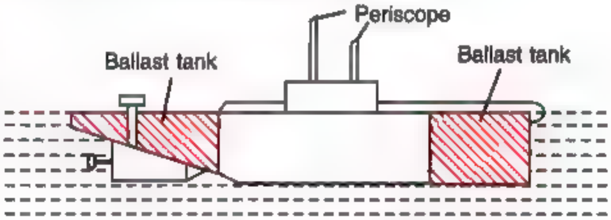

Fig. 5.13 Submarine

A submarine can be made to dive into the water or rise up to the surface of water as and when desired. If a submarine is to dive, its ballast tanks are filled with water so that the average density of submarine becomes greater than the density of sea water and the submarine dives into the water. If submarine is to rise, water from the ballast tanks is forced out into the sea by allowing the compressed air to enter the tank. This makes the average density of submarine less than that of sea water. As a result, the weight of water displaced by its partially small submerged part becomes equal to the weight of submarine and hence it rises up to the surface of water.

(iv) *Floatation of iceberg*

The density of ice is less than the density of water. The density of ice is 0·917 g cm^{-3} and that of water is 1 g cm^{-3}. Therefore, huge masses of ice known as *icebergs* are able to float on water with their major part inside the water surface and only a small portion above the water surface.

Volume of iceberg above the water surface while floating : If the total volume of an iceberg is V and the volume of iceberg submerged is v, then by the principle of floatation,

Weight of water displaced by the submerged part of iceberg = Total weight of iceberg

or $$v \times \rho_{water} \times g = V \times \rho_{ice} \times g$$

or $$\boxed{\frac{v}{V} = \frac{\rho_{ice}}{\rho_{water}}}$$ ....(5.20)

Examples : (1) An iceberg ($\rho_{ice} = 0{\cdot}917$ g cm^{-3}) floats on water (ρ_{water} = 1·0 cm^{-3}) with volume $v = 0{\cdot}917\ V$ *i.e.* 91·7% of its total volume below the water surface or only 8·3% of its volume above the water surface as shown in Fig. 5.14.

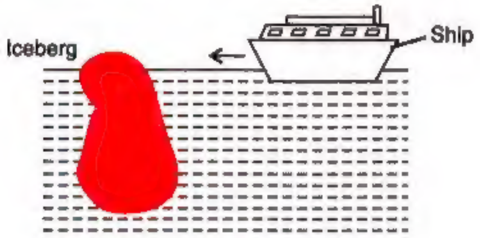

Fig. 5.14 Floating iceberg

(2) An iceberg ($\rho_{ice} = 0{\cdot}917$ g cm^{-3}) floats on sea water ($\rho_{water} = 1{\cdot}026$ g cm^{-3}) with volume $v = \frac{0{\cdot}917}{1{\cdot}026} V = 0{\cdot}893\ V$ *i.e.*, 89·3% of its total volume submerged inside sea water and only 10·7% above sea water.

Icebergs are dangerous for ships : Icebergs being lighter than water, float on water with their major part (nearly 90%) inside water and only a small part (≈10%) outside water. Since portion of iceberg inside the water surface depends upon the density of sea water, therefore for the driver of ship, it becomes difficult to estimate the size of iceberg. Thus an iceberg is very dangerous for the ship as it may collide with the ship and cause damage.

No change in level of water on melting a floating piece of ice : When a floating piece *of ice melts into water, it contracts by the* volume equal to the volume of ice piece above the water surface while floating on it. Hence the level of water does not change when the ice floating on it melts.

(v) *Floatation of fish*

Many species of fish have an organ called a *swim bladder.* It acts like the ballast (or floatation) tank of a submarine. When a fish has to rise up in water, it diffuses gas from its fluid into the bladder, so its volume increases and its average density decreases. This increases the volume of water displaced by the fish and so the upthrust on fish increases due to which it rises up. When the fish has to come down, it empties its bladder to the required extent, so its volume decreases and density increases. Hence upthrust on fish decreases and it sinks in water.

(vi) *Rising of balloons*

When a light gas like hydrogen or helium (density much less than that of air) is filled in a balloon, the weight of air displaced by the inflated balloon (*i.e.*, upthrust) becomes more than the weight of the gas filled balloon and it rises up.

The balloon does not rise indefinitely. The reason is that the density of air decreases with altitude. Therefore as the balloon gradually goes up, the weight of the displaced air (*i.e.*, upthrust) decreases. It keeps on rising as long as the upthrust on it exceeds its weight. When upthrust becomes equal to its weight, it stops rising further.

EXAMPLES

1. A block of wood of volume 25 cm^3 floats on water with 20 cm^3 of its volume immersed. Calculate : (i) the density, and (ii) the weight of block of wood.

Given : Volume of block $V = 25$ cm^3,
Volume immersed in water $v = 20$ cm^3

(i) If density of wood is ρ g cm^{-3}, then by principle of floatation.

Weight of block of wood = Weight of water displaced by the immersed part of block.

$V\rho g = v \times 1 \times g$ (since density of water = 1 g cm^{-3})

$\therefore\ \rho = \frac{v}{V} = \frac{20}{25} = \mathbf{0{\cdot}8\ g\ cm^{-3}}$

(ii) Weight of block of wood $= V\rho g$
$= 25 \times 0{\cdot}8 \times g = 20\,g$ dyne
$= \mathbf{20\ gf}$.

2. A block of iron floats on mercury. Find the fraction of volume which remains immersed in mercury. (Densities of iron and mercury are $7{\cdot}8$ g cm^{-3} and $13{\cdot}6$ g cm^{-3} respectively)

Let V be the volume of iron block and v be its volume immersed in mercury. For floatation,

Weight of block = Weight of mercury displaced by the immersed portion of block.

i.e., $V \times 7{\cdot}8 \times g = v \times 13{\cdot}6 \times g$

or $\frac{v}{V} = \frac{7.8}{13.6} = \mathbf{0{\cdot}574}$

or $v = 0{\cdot}574\ V$

The fraction **0·574 of total volume** will remain immersed in mercury.

3. An iceberg floats on fresh water with a part of it outside the water surface. Calculate the fraction of the volume of the iceberg which is below the water surface.

Given : density of ice = 917 kg m^{-3}, density of fresh water = 1000 kg m^{-3}.

By the principle of floatation,

$$\frac{\text{Volume of iceberg immersed}}{\text{Total volume of iceberg}} = \frac{\text{Density of ice}}{\text{Density of fresh water}}$$

$$= \frac{917}{1000} = \mathbf{0{\cdot}917}$$

Thus $0{\cdot}917^{th}$ part of volume of iceberg will remain below the water surface.

4. A block of wood floats on water with $\frac{2}{5}$th of its volume above the water surface. Calculate the density of wood.

Let the volume of block be V and density of wood be ρ. Volume of block above the surface of water $= \frac{2}{5}\ V$.

$\therefore$ Volume of block immersed $v = V - \frac{2}{5}\ V = \frac{3}{5}\ V$

By the principle of floatation,

Weight of the block = Weight of water displaced by the immersed part of block

i.e., $V \times \rho \times g = \frac{3}{5}\ V \times 1 \times g$

$\rho = \frac{3}{5} = \mathbf{0{\cdot}6\ g\ cm^{-3}}$.

(Here the density of water is taken as 1 g cm^{-3})

5. A piece of wood of volume 200 cm^3 and density $0{\cdot}84$ g cm^{-3} floats in a liquid of density $1{\cdot}05$ g cm^{-3}.

(i) What volume of wood will remain above the surface of liquid ?

(ii) What force must be exerted on wood to keep it totally submerged ?

Given, $V = 200\ cm^3$, $\rho_s = 0{\cdot}84$ g cm^{-3}, $\rho_L = 1{\cdot}05$ g cm^{-3}

(i) Let V' cm^3 be the volume of wood which remains above the surface of liquid. Then submerged volume of wood $v = V - V' = (200 - V')\ cm^3$ and by the principle of floatation,

Weight of wood piece

= Upthrust due to submerged part of wood

$200 \times 0{\cdot}84 \times g = (200 - V') \times 1{\cdot}05 \times g$

$\therefore \quad V' = \frac{200 \times (1{\cdot}05 - 0{\cdot}84)}{1{\cdot}05}$

$= \frac{200 \times 0{\cdot}21}{1{\cdot}05} = \mathbf{40\ cm^3}$

(ii) When wood piece is totally submerged, then

Upthrust $= V \times \rho_L \times g = 200 \times 1{\cdot}05 \times g$

$= 210$ gf (upwards)

Weight of wood piece $= V \times \rho_s \times g = 200 \times 0{\cdot}84 \times g$

$= 168$ gf (downwards)

$\therefore$ Force to be exerted to keep the wood totally submerged

= Upthrust – Weight of wood piece

= 210 – 168 = **42 gf.**

6. The volume of a balloon is 1000 m^3. It is filled with helium of density $0{\cdot}18$ kg m^{-3}. What maximum load can it lift ? Density of air is $1{\cdot}29$ kg m^{-3}.

Given, volume of balloon $V = 1000\ m^3$,

density of helium $\rho = 0{\cdot}18$ kg m^{-3},

density of air = $1{\cdot}29$ kg m^{-3}

Weight of helium filled balloon

$= V \times \rho \times g = 1000 \times 0{\cdot}18 \times g$

$= 180\ g$ N = 180 kgf

Weight of air displaced = upthrust

$= V \times$ density of air $\times g$

$= 1000 \times 1{\cdot}29 \times g$

$= 1290\ g$ N = 1290 kgf

Resultant upward force on balloon

= upthrust – weight of balloon

= 1290 – 180 = 1110 kgf.

So it can lift a maximum load of **1110 kgf.**

1. State the principle of floatation.
2. A body is held immersed in a liquid. (i) Name the *two* forces acting on body and draw a diagram to show these forces. (ii) State how do the magnitudes of two forces mentioned in part (i) determine whether the body will float or sink in liquid when it is released. (iii) What is the net force on body if it (a) sinks, (b) floats ?
3. When a piece of wood is suspended from the hook of a spring balance, it reads 70 gf. The wood is now lowered into water. What reading do you expect on the scale of spring balance ?
   [**Hint :** The piece of wood will float on water and while floating, apparent weight = 0]. **Ans.** Zero
4. A solid iron ball of mass 500 g is dropped in mercury contained in a beaker. (a) Will the ball float or sink ? Give reason. (b) What will be the apparent weight of ball ? Give reason.
   Ans. (a) Float, **Reason:** Density of ball (*i.e.*, iron) is less than the density of mercury. (b) Zero
   Reason : While floating, upthrust = weight.
5. How does the density of a substance determine whether a solid piece of that substance will float or sink in a given liquid ?
   Ans. The body will float if $\rho_s \leq \rho_L$ and it will sink if $\rho_s > \rho_L$
6. Explain why an iron nail floats on mercury, but it sinks in water.
   [**Hint :** Density of iron is less than that of mercury, but more than that of water]
7. A body floats in a liquid with a part of it submerged inside liquid. Is the weight of floating body greater than, equal to or less than upthrust ?
   Ans. Equal to
8. A homogeneous block floats on water (a) partly immersed (b) completely immersed. In each case state the position of centre of buoyancy *B* with respect to the centre of gravity *G* of the block.
   Ans. (a) *B* will lie vertically below *G* (b) *B* will coincide *G*
9. Fig. 5.15 shows the same block of wood floating in three different liquids *A*, *B* and *C* of densities ρ_1, ρ_2 and ρ_3 respectively. Which of the liquid has the highest density ? Give reason for your answer.

Fig. 5.15

Ans. *C*

Reason : The upthrust on the body by each liquid is same and it is equal to the weight of body. But upthrust = volume submerged × ρ_L × g. For liquid *C*, since volume submerged is least so density ρ_3 must be maximum.

10. Draw a diagram to show the forces acting on a body floating in water with its some part submerged. Name the forces and show their points of application. How is the weight of water displaced by the floating body related to the weight of the body itself ?
11. What is centre of buoyancy ? State its position for a floating body with respect to the centre of gravity of body.
12. A balloon filled with helium gas floats in a big closed jar which is connected to an evacuating pump. What will be your observation, if air from jar is pumped out ? Explain your answer.
    Ans. Observation : The balloon will sink.
    Explanation : As air is pumped out from jar, the density of air in jar decreases, so the upthrust on balloon decreases. As weight of balloon exceeds the upthrust on it, it sinks.
13. A block of wood is so loaded that it just floats in water at room temperature. What change will occur in the state of floatation, if
    (a) some salt is added to water,
    (b) water is heated ?
    Give reason.
    Ans. (a) Floats with some part outside water.
    Reason : On adding some salt to water, the density of water increases, so upthrust on block of wood increases and hence the block rises up till the weight of salty water displaced by the submerged part of block becomes equal to the weight of block. (b) Sinks.
    Reason : On heating, the density of water decreases, so upthrust on block decreases and weight of block exceeds the upthrust due to which it sinks.
14. A body of volume *V* and density ρ_s, floats with volume *v* inside a liquid of density ρ_L. Show that $\frac{v}{V} = \frac{\rho_s}{\rho_L}$.
15. Why is the floating ice less submerged in brine than in water ?
    Ans. Density of brine is more than the density of water.
16. A man first swims in sea water and then in river water.
    (i) Compare the weights of sea water and river water displaced by him.
    (ii) Where does he find it easier to swim and why?
    Ans. (i) 1 : 1 (in each case the weight of water displaced will be equal to the weight of man) (ii) In sea water because the density of sea water is more than that of river water so his weight is balanced in sea water with his less part submerged inside it.

17. An iron nail sinks in water while an iron ship floats on water. Explain the reason.

18. What can you say about the average density of a ship floating on water in relation to the density of water ?

Ans. Average density of ship is less than the density of water.

19. A piece of ice floating in a glass of water melts, but the level of water in glass does not change. Give reason.
[**Hint** : Ice contracts on melting.]

20. A buoy is held inside water contained in a vessel by tying it with a thread to the base of the vessel. Name the *three* forces that keep the buoy in equilibrium and state the direction in which each force acts.

Ans. (i) Weight of buoy vertically downwards, (ii) upthrust of water on buoy vertically upwards, and (iii) tension in thread vertically downwards.

21. A loaded cargo ship sails from sea water to river water ? State and explain your observation.

22. Explain the following :
    (a) Icebergs floating in sea are dangerous for ships.
    (b) An egg sinks in fresh water, but floats in a strong salt solution.
    (c) A toy balloon filled with hydrogen rises to the ceiling, but if filled with carbon dioxide sinks to the floor.
    (d) As a ship in harbour is being unloaded, it slowly rises higher in water.
    (e) A balloon filled with hydrogen rises to a certain height and then stops rising further.
    (f) A ship submerges more as it sails from sea water to river water.

Multiple choice type :

1. For a floating body, its weight W and upthrust F_B on it are related as :
   (a) $W > F_B$ (b) $W < F_B$
   (c) $W = F_B$ (d) nothing can be said.
   Ans. (c) $W = F_B$

2. A body of weight W is floating in a liquid. Its apparent weight will be :
   (a) equal to W (b) less than W
   (c) greater than W (d) zero. Ans. (d) zero

3. A body floats in a liquid A of density ρ_1 with a part of it submerged inside liquid while in liquid B of density ρ_2 totally submerged inside liquid. The densities ρ_1 and ρ_2 are related as :
   (a) $\rho_1 = \rho_2$ (b) $\rho_1 < \rho_2$
   (c) $\rho_1 > \rho_2$ (d) nothing can be said
   Ans. (c) $\rho_1 > \rho_2$

Numericals :

1. A rubber ball floats on water with its 1/3rd volume outside water. What is the density of rubber ?
   Ans. 667 kg m^{-3}

2. A block of wood of mass 24 kg floats on water. The volume of wood is 0·032 m^3. Find :
   (a) the volume of block below the surface of water,
   (b) the density of wood.
   (Density of water = 1000 kg m^{-3})
   Ans. (a) 0·024 m^3 (b) $7{\cdot}5 \times 10^2$ kg m^{-3}

3. A wooden cube of side 10 cm has mass 700 g. What part of it remains above the water surface while floating vertically on the water surface ?
   Ans. 3 cm height

4. A piece of wax floats on brine. What fraction of its volume is immersed ?
   Density of wax = 0·95 g cm^{-3}, Density of brine = 1·1 g cm^{-3}. Ans. 0·86

5. If the density of ice is 0·9 g cm^{-3}, what portion of an iceberg will remain below the surface of water in a sea ? (Density of sea water = 1·1 g cm^{-3})
   Ans. $\frac{9}{11}$th (or 0·818th) part.

6. A piece of wood of uniform cross section and height 15 cm floats vertically with its height 10 cm in water and 12 cm in spirit. Find the density of (i) wood and (ii) spirit.
   Ans. (i) 0·667 g cm^{-3}, (ii) 0·833 g cm^{-3}

7. A wooden block floats in water with two-third of its volume submerged. (a) Calculate the density of wood. (b) When the same block is placed on oil, three-quarter of its volume is immersed in oil. Calculate the density of oil.
   Ans. (a) 667 kg m^{-3}, (b) 889 kg m^{-3}

8. The density of ice is 0·92 g cm^{-3} and that of sea water is 1·025 g cm^{-3}. Find the total volume of an iceberg which floats with its volume 800 cm^3 above water. Ans. 7809·5 cm^3

9. A weather forecasting plastic balloon of volume 15 m^3 contains hydrogen of density 0·09 kg m^{-3}. The volume of an equipment carried by the balloon is negligible compared to its own volume. The mass of empty balloon alone is 7·15 kg. The balloon is floating in air of density 1·3 kg m^{-3}. Calculate : (i) the mass of hydrogen in the balloon, (ii) the mass of hydrogen and balloon, (iii) the total mass of hydrogen, balloon and equipment if the mass of equipment is x kg, (iv) the mass of air displaced by balloon and (v) the mass of equipment using the law of floatation.
   Ans. (i) 1·35 kg (ii) 8·5 kg (iii) $(8{\cdot}5 + x)$ kg (iv) 19·5 kg (v) 11 kg